Printing Out Trees: Toward the Design of Tangible Objects for Education

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Abstract.
When educational technologists employ a term like “scientific visualization”, what they usually mean is that scientific concepts or information are portrayed creatively on a two-dimensional screen. Increasingly, however, the advent of novel fabrication devices enables computers to “print out” physical objects that can themselves serve as models or representations of scientific concepts. This paper describes a working prototype of an educational application, entitled Growth, that enables users to print physical models of trees (and other botanical forms) with the aid of a 3D prototyping device. We describe the implementation of the program (and the mathematical theory of “L-systems” that underlies its biological content); and we show several examples of “printed-out trees” in plaster created with the software. The paper ends by arguing that Growth is an early instance of a style of educational software that is likely to burgeon in importance and variety during the coming decade.

Keywords
Computers and education; three-dimensional printing; simulation and visualization; biological modeling; L-systems.

Introduction: Creating Tangible Objects for Education
The notion of “scientific visualization” has traditionally been associated with the use of the computer screen as a viewing device. In this tradition, to make scientific data or ideas “visualizable” is tantamount to finding some creative way of representing or rendering them in a necessarily two-dimensional medium. Often, this practice involves the creation of “virtual” three-dimensional forms—that is, spatial forms that can be portrayed on a screen so as to convey an illusion of true three-dimensional structure. And going further, there are more elaborate virtual reality systems that employ (e.g.) special headsets or glasses to provide a still richer representation of three-dimensional objects and scenes.

A standard assumption of all these techniques is that the form being visualized is a “virtual” object that exists only in some ethereal, computational form. Or, to put it in more mundane terms: once the computer is switched off, the object is “gone” and no longer visualizable. Undoubtedly, there have been tremendous advances in this tradition of scientific visualization, and the viewing of virtual objects will remain a central element of educational (and scientific) computing in the foreseeable future. Nonetheless, virtual objects have their own limitations: and there are, or ought to be, alternatives to this traditional interpretation of scientific visualization.

Increasingly, the advent of powerful, accessible fabrication devices suggests such an alternative. Rather than exalting the computer screen as the primary output device for computational data (with the printer as a vastly underestimated sidekick), novel devices such as milling machines, laser cutters, computer-controlled sewing machines, and so forth now enable users to transform a wide variety of virtual objects into physical form. In this paper, we focus on the use of a 3D prototyper (or “3D printer”, as it is sometimes called) as an output device for educational computing. Briefly—we’ll go into more detail shortly—a 3D printer is a device that takes a specification of a solid object and “prints out” that object in a material such as plaster, plastic, or metal. Such devices have been used in recent years within industrial settings, to produce (e.g.) physical prototypes of cars or appliances; in the near future, however, 3D printers are likely to become increasingly affordable for settings such as schools and community centers.

This paper describes a working prototype of a computer application called Growth that makes use of the 3D printer as an output device. The basic idea behind Growth is that it permits student to create models of trees (and other
botanical forms) via the well-known mathematical technique of employing \textit{L-systems}. \textit{L-systems} can be thought of as “geometric grammars” whose rules can be tailored to specify recursive structures such as trees, vines, and ferns. In the Growth system, students employ \textit{L-systems} to model a huge variety of tree-like forms which can be viewed on the computer screen; but importantly, these forms may then be translated into a file format suitable for printing out as tangible models from a 3D printer.

Figure 1 shows a tree that was designed and printed using the Growth application. The tree has been rendered in plaster (the stand supporting the tree was created separately). Conceivably the tree could now be further decorated (e.g. by painting, or by “dressing” the branches with paper leaves); but in any event, it should be clear that by adding three-dimensional objects to two-dimensional screen renderings, educational computing can take advantage of a powerful new means of “scientific visualization”.

![Figure 1](image1.png)

Figure 1. A tangible model of a tree printed with the Growth system.

In the remainder of this paper, we describe the Growth application in greater detail; and we use this particular application as a springboard for a more wide-ranging discussion of the role of fabrication devices in educational computing more generally. The following (second) section gives an overview of Growth, and provides a brief technical introduction to the subject of \textit{L-systems} as mathematical representations of botanical forms. In the third section we explore the advantages and disadvantages of tangible output (in comparison to screen-based output) for scientific visualization and education. The final (fourth) section discusses related work in the area of “educational fabrication”, and describes plausible directions for future development of Growth and related applications.

\textbf{Growth: An Overview and Scenario}

The Growth application was created to exemplify the way in which an interactive educational system can make use of novel fabrication techniques. While Growth is affirmatively a work-in-progress (it could benefit from many potential additions both in code and documentation), it is nonetheless a self-contained working system. Growth has been implemented in Java, and runs on both PC and Macintosh computers.

![Figure 2](image2.png)

Figure 2. A screenshot of the Growth system in use. The tree being designed here is based on the same \textit{L-system} as the printed version shown in Figure 1 earlier.

The basic idea behind Growth is suggested by the screenshot of the application shown in Figure 2. The user begins by selecting (from a menu) one of a set of available “\textit{L-systems}” that correspond to basic tree types. (We will explain the theory behind \textit{L-systems} later in this section.) Once a system is selected, the user is provided with a set of controls (shown at the right of the screen in Figure 2) that serve to specify the parameters of the particular tree to be created. In Figure 2, the user has selected a set of parameters for a given \textit{L-system} (the “Aono” system), and the software produces a three-dimensional rendering of the desired tree in the panel toward the left of the screen. By using the mouse, the user can now view this newly-created tree from any desired angle.
By exploring choices of parameters, the user can try out an immense range of distinct variations for any single L-system. Once the user has selected the parameters for the tree that she wishes to create, she may now translate that tree into printable form by selecting a menu choice in the application. This choice creates a representation of the tree in a standard format (STL format) suitable for output from a 3D printer. The tree may now be printed to produce a physical tree model like the one shown in Figure 1.

In the remainder of this section, we expand on several elements of this brief description: L-systems as a representation of botanical forms, the basic hardware and software aspects of 3D printing, and several additional interface features of the Growth application.

**L-Systems: A Brief Introduction**

L-systems (or Lindenmayer systems, as they are known, after their inventor Aristid Lindenmayer) are a well-known mathematical formalism for representing recursive forms in nature. Detailed descriptions of L-systems can be found in [5]; but for the purposes of this paper, they can be characterized as algorithmic representations similar in spirit to context-free grammars. The basic idea behind L-systems (as behind context-free grammars) is that a set of “expansion rules” is used to generate complex forms from a simple initial state. In traditional context-free grammars, the rules often correspond to the expansion of (e.g.) complete sentence structures based on grammars for natural languages; in L-systems, the rules can be interpreted as geometric rules that generate branched structures. The L-systems of Growth are in fact based on geometric expansion rules that correspond to moves of a 3-dimensional Logo-style “turtle”. [Cf. 1]

An example may help to clarify this discussion. Suppose we create an abstract context-free grammar (of five rules) intended to represent a generic tree form:

1. **TREE → STEM LEFT-BRANCH RIGHT-BRANCH**
2. **TREE → STEM**
3. **STEM → MOVE-FORWARD**
4. **LEFT-BRANCH → LEFT-TURN TREE**
5. **RIGHT-BRANCH → RIGHT-TURN TREE**

The basic idea behind this grammar is that it represents the recursive structure of a tree. Interpreting the rules as dictating moves of the three-dimensional turtle, we could rewrite this grammar as a program:

Rule 1 (TREE): To draw a tree, first execute the STEM rule; then execute the LEFT-BRANCH and RIGHT-BRANCH rules.

Rule 2 (TREE): Alternatively, to draw a tree, simply execute the STEM rule.

Rule 3 (STEM): Move the turtle forward, drawing a stem.

Rule 4 (LEFT-BRANCH): Turn the turtle along a given tree-specific left-turn angle; then execute the TREE rule.

Rule 5 (RIGHT-BRANCH): Turn the turtle along a tree-specific right-turn angle; then execute the TREE rule.

This five-rule grammar is represented graphically in Figure 3. In the figure, we have drawn a three-level tree according to the specified rules. (It may be also worth noting that the left-turn and right-turn angles represent both “pitch” and “roll” turns for the 3D turtle.)

![Figure 3](image.png)
3D Printers: Hardware and Software

In the Growth system, once a suitable tree has been created, it may be translated to a form suitable for output to a 3D printer. In this section, we outline the basic ideas behind 3D printers, and how a printable 3D format is created.

There are by now a variety of commercially available 3D printers. In general, these printers operate by laying down successive “layers” of a desired three-dimensional form in some particular material. Thus, to generate (for instance) a sphere, a 3D printer would begin by laying down a tiny circle; then successively larger circles, up to a great circle (whose circumference represents the equator) of the sphere; then successively smaller circles.

Different printers employ different materials for this purpose: for example, a plastic printer might at each step lay down a liquid plastic that is quickly “cured” via ultraviolet light. The particular printer in our own lab is a Z-System 300 printer [W2] that employs plaster as its printing material: at each step of the printing process, a layer of plaster is swept into a “print volume” and a liquid binder is used to harden just the desired cross-section of plaster for this particular layer. Once a complete object has been created in this printer, it is embedded in a volume of loose powder which must be swept away in order to retrieve the object. (This is often the trickiest step in retrieving a delicate or detailed form such as a model of a tree.)

The most general-purpose representation for 3D models is known as the STL (for "stereolithography") format. In STL, the surface of a closed solid is represented as a set of triangles. For instance, a cube could easily be represented as a set of twelve right triangles (two per square face). There are additional constraints that must be accounted for in STL format: for instance, each triangle must be accompanied by a normal vector pointing outward from the solid shape’s surface, and each triangle must share one complete edge with exactly one other triangle.

A tree in Growth is represented as a collection of cylindrical forms corresponding to the various branches. While it would be possible in principle to represent a Growth tree as one single (highly complex) solid form, a collection of distinct cylinders may be used as long as the set-union of those cylinders corresponds to the desired tree. To illustrate, the simple tree diagrammed in

Figures 4 (top) and 5 (bottom): Two more trees designed in Growth and printed out in tangible form.

Space considerations do not permit a thorough discussion of L-systems and their uses; but two additional points deserve mention here. First, virtually any particular L-system (such as the simple one described above) implicitly includes at least several numerical parameters that can be altered to produce varying tree structures. For instance, the L-system of Figure 3 can be “tuned” by setting particular values of the left-turn and right-turn angles; or, alternatively, by allowing each invocation of the STEM rule to set the length of the drawn stem according to the current level of the tree. Thus, even for one particular choice of L-system rules, a wide variety of distinct trees may be drawn. Second, by changing or adding to the rules of the L-system itself, one can generate new “types” of trees. Figures 4 and 5 show trees designed in Growth based on two L-systems (the “Honda” and “Singleton” systems) different from the one shown in Figure 1; these have a substantially different appearance from the type of tree shown in Figure 1.
Figure 3 earlier could be represented by seven distinct (overlapping) STL cylinders, rather than one single complex branched solid form. It should be mentioned that (depending on the accompanying software) not all 3D printers are currently capable of printing such a collection of overlapping cylinders as a single object.

Additional features of the Growth system

Beyond those features of the system already described here, there are several others of interest. At present, Growth includes four basic L-systems from which to choose; each L-system is accompanied by a set of interface tools for adjusting the parameters of the given system. In addition, there is documentation geared toward “power users” (e.g., teachers) that allows those users to add new L-systems and related interface tools to the program. When a new L-system is installed, it is added to the menu of choices in Growth, and the accompanying interface tools allow all users to explore the new system in the same manner as the original four. Thus, a teacher (or expert student) who wishes to try out a brand new L-system should be able to do so.

There are also graphical menu choices in the system (e.g., for choosing the color of displayed trees on the screen). One especially useful choice allows the user to “tune” the approximation to a cylinder according to the number of sides in the base: for instance, approximating each cylinder base with an octagon leads to “cylinders” which are in fact octagonal prisms; the resulting tree, when drawn on the screen, looks rather blocky. A “cylinder” with a 24-sided base will cause the resulting tree to appear a good deal more smooth and natural. The trade-off, unsurprisingly, is that the more natural tree takes somewhat longer for the system to calculate; thus, a user who wishes to see very rapid approximations to a tree under design can quickly explore “blocky” trees; when she arrives at the perfect design, she can give the structure a good deal more detail before printing.

Three-Dimensional Printing in Education: the Good, the Bad, and the Possible

The motivation behind this project is to provide an illustration of the remarkable educational potential of personal fabrication tools for educational computing. In our view, the ability to “print out” an object such as a tree, without much more travail than printing out a document, could well revolutionize the landscape of children’s activities. We will expand on this view shortly, but first it is useful to discuss the advantages and (at least current) disadvantages of three-dimensional fabrication in educational settings.

For most educational technologists, the comparison that springs to mind is that between a physical object (like a printed tree) and a screen-based rendering (like those shown in the screen interface of Growth). Obviously, both these representations have their strengths and weaknesses, but the advantages of tangibility deserve a little attention here. Three-dimensional objects can be handled at the same time that they are observed, thus integrating physical and visual input: it is possible to touch a printed tree and to feel its branching structure in a way that is not possible with purely screen-based renderings. The virtual-reality enthusiast may at this point interject that it should be possible (with special apparatus) to “feel” even a virtual rendering of a tree; but of course a physical object needs no special apparatus for this purpose. Physical objects are, almost without conscious attention, handed from one student to another, observed by multiple students simultaneously, or taken home for the evening.

Moreover, there are interesting and subtle social affordances of physical objects, particularly relevant to educational settings. A printed tree can be put on the shelf and displayed for weeks or months as an example of student work; or a large number of trees may be printed out to compose student-designed “forests” and dioramas. A student might design a printed object for a parent or friend as a gift; or he might keep an object as a souvenir of a class. All these “social” aspects of physical objects may seem rather homely, but in fact they reflect the emotional pull of physical things. (Cf. [2].) For reasons that are interesting to speculate on, students seldom have this sort of attachment to virtual objects: it is the rare simulation or program file that is given as a gift or treated as a souvenir.

Undeniably, there are profound limitations of tangible objects as well, relative to their “virtual” counterparts. One cannot instantly change the color or scale of a physical object, or endow it with animation, or “zoom into” its interior, among a myriad other possibilities. Our own view is that the roles of virtual and tangible objects can be mutually supporting and
complementary; one needn’t pick one or the other. The Growth program, for example, is intended to combine at least some of the affordances of virtual trees (e.g., the ability to turn it on the screen, the ability to change color) with those of physical objects.

At present, the use of 3D printers in education may still be rightly described as embryonic. Our own printer, though relatively inexpensive in contrast to similar devices, is still too expensive for most schools; and the materials used (plaster, binder, and an “infiltration” coat that must be applied after removing the plaster object from the printer) are prohibitively messy and hazardous for use by unsupervised students. We believe that the state of the art in 3D printing is likely to shift in the coming decade so that such devices will become more accessible, safe, and affordable for school settings. This is hardly a remarkable prediction: the past several decades have seen similar striking developments in (e.g.) computers, printers, disk drives, and many other technological artifacts that were once thought inaccessible in school settings.

Printing trees or botanical forms is of course only one instance of the remarkable potential of 3D fabrication in education. Students could, for instance, make sets of personalized construction kit pieces; they could populate their printed-tree dioramas with printed animal models; they could make accurate model trains, cars, or airplanes; they could make delicate and complex mathematical models (e.g., classical polyhedra or curved surfaces); they could make accurate historical models (e.g., to create displays of Stonehenge or the Acropolis). Again, we feel that the role of fabrication tools for students is just beginning to be felt, but will eventually be pervasive; and, at its best, the use of such tools will permit students a tremendous range of creativity and expressiveness.

**Related and Ongoing Work: Future Directions**

The most direct influence upon this work has been from the efforts of researchers at the MIT Media Lab-particularly Mitchel Resnick and Hiroshi Ishii (and their respective colleagues) in blending tangible and computational media for educational purposes. (Cf. [4], [6].) For the most part, these efforts focus on embedding computational capabilities inside physical objects—a subject of interest to us as well—but as the discussion of fabrication in Neil Gershenfeld’s book *When Things Start to Think* [3] makes clear, fabrication tools represent another major focus of interest for the Media Lab as an institution.

In the near term, we intend to fine-tune the interface for the Growth system (and to add at least one or two more L-system choices); and we plan to make the system available as freeware for downloading before the end of the year. [W1] In the longer-term future, we hope to develop a variety of similar tools (e.g., for the design of mathematical surfaces or animal forms, as mentioned earlier) that build on the insight and experiences gained through this initial foray into the educational role of fabrication tools.

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**References**


**Websites:**

[W1] Craft Technology Laboratory at U. of Colorado: www.cs.colorado.edu/~ctg